

Decaying Λ cosmology with varying G

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Abstract

We study a uniform and isotropic cosmology with a decaying vacuum energy density, in the realm of a model with a time varying gravitational “constant”. We show that, for late times, such a cosmology is in accordance with the observed values of the cosmological parameters. In particular, we can obtain the observed ratio between the matter density and the total energy density, with no necessity of any fine tuning.

I. INTRODUCTION

The advent of precise observational cosmology and the recent discovery of a small cosmological constant Λ have reinforced two theoretical problems that have remained open throughout the years [1]. The first is the absolute value of Λ , 122 orders of magnitude smaller than the value expected for the Planck time. The second is why the vacuum energy density has, at the present time, the same order as the matter density. This second problem is usually known as the cosmic coincidence problem.

A possible explanation for the small value of Λ can be based on the idea that the vacuum energy density is not constant, but decays as the universe expands [2]-[7]. Nevertheless, this does not explain the cosmic coincidence, except by means of a fine tuning of initial conditions.

In the present contribution, we will consider a FLRW, spatially flat, decaying Λ cosmology, in the realm of a model where the gravitation “constant” G varies with time at a cosmological scale. We will show that, in the asymptotic limit of late times, our model admits, besides the usual de Sitter solution, three other solutions characterized by a constant ratio between the matter density and the total energy density. Two of them have a decelerating expansion. The third one presents a coasting expansion, with the universe age given by $Ht = 1$, and a relative matter density given by $\rho_m/\rho = 1/3$.

II. DECAYING Λ SOLUTIONS WITH CONSTANT G

In the flat case, the Einstein equations are given by

$$\rho = 3H^2, \tag{1}$$

$$\dot{\rho} + 3H(\rho + p) = 0, \tag{2}$$

where $H = \dot{a}/a$ is the Hubble parameter.

We will consider a twofold energy content, formed by dust matter with energy density ρ_m , and by a vacuum term with equation of state $p_\Lambda = -\rho_\Lambda$. Then we have, for the total energy and pressure,

$$\rho = \rho_m + \rho_\Lambda, \tag{3}$$

$$p = -\rho_\Lambda. \tag{4}$$

Substituting (1), (3) and (4) into (2), we obtain

$$2\dot{H} + \rho_m = 0. \quad (5)$$

If we suppose that, for late times, H and ρ_m falls monotonically with the scale factor a , we can expand them in negative power series of a . Taking the dominant terms, we then have

$$H = \beta/a^k, \quad (6)$$

$$\rho_m = \gamma/a^n, \quad (7)$$

where n and k are positive integers.

Substituting into (5) leads to

$$\gamma a^{-n} - 2k\beta^2 a^{-2k} = 0. \quad (8)$$

For the above equation to be valid for any (large) value of a , we must have

$$n = 2k, \quad (9)$$

$$\gamma = 2\beta^2 k. \quad (10)$$

Now, it is straightforward to show that

$$\Omega_m \equiv \rho_m/\rho = 2k/3. \quad (11)$$

Since $\Omega_m \leq 1$, it follows that $k = 0, 1$. In the case $k = 0$, we have

$$H = \beta, \quad (12)$$

$$\rho_m = 0. \quad (13)$$

This solution corresponds to a de Sitter universe, with a constant vacuum energy density.

In the case $k = 1$, it follows that

$$a = \beta t, \quad (14)$$

$$Ht = 1, \quad (15)$$

$$\rho_m/\rho = 2/3. \quad (16)$$

As discussed in [8, 9], among the solutions of the form $a \propto t^n$, the best fitting of supernova and radio sources observations is obtained in the coasting case, $a \propto t$. On the other hand, the relation $Ht = 1$ gives the best estimation for the universe age. Nevertheless, we know that a relative matter density equal to $2/3$ is not consistent with the observed amount of visible and dark matter.

III. DECAYING Λ SOLUTIONS WITH VARYING G

In the realm of a varying gravitational “constant”, we will still consider the ansatz

$$\rho = \frac{3H^2}{8\pi G}, \quad (17)$$

$$\dot{\rho} + 3H(\rho + p) = 0, \quad (18)$$

where we have re-introduced the factor $8\pi G$.

The continuity equation (18) does not depend on the varying or constant character of G , being just an expression of energy conservation. As far as relation (17) is concerned, it is valid today, and we will assume that it is valid for any late time.

As to the variation law for G , let us take the Eddington-Weinberg empirical relation [10]

$$G \approx \frac{H}{m^3} = \frac{H}{8\pi\lambda}, \quad (19)$$

where m has the order of the pion mass, and the constant λ was introduced for convenience. Once again, this relation is valid today, and we will assume that it is valid for any late time.

Substituting (19) into (17), we have

$$\rho = 3\lambda H. \quad (20)$$

For the total energy density and pressure, we will use again

$$\rho = \rho_m + \rho_\Lambda, \quad (21)$$

$$p = -\rho_\Lambda. \quad (22)$$

Carrying (20-22) into the continuity equation (18), we obtain

$$\lambda\dot{H} + \rho_m H = 0. \quad (23)$$

Let us expand again H and ρ_m in negative powers of a , and take the leading terms

$$H = \beta/a^k, \quad (24)$$

$$\rho_m = \gamma/a^n. \quad (25)$$

Substituting into (23) leads to

$$\gamma a^{-n} - \lambda\beta k a^{-k} = 0. \quad (26)$$

Now, the possible solutions are given by

$$k = n, \quad (27)$$

$$\gamma = \lambda\beta n. \quad (28)$$

It is easy to show that this leads to

$$\rho_m/\rho = n/3, \quad (29)$$

which means that, now, $n = 0, 1, 2, 3$.

For $n = 0$, we have

$$H = \beta, \quad (30)$$

$$\rho_m = 0, \quad (31)$$

$$G = \frac{\beta}{8\pi\lambda}. \quad (32)$$

That is, the usual de Sitter solution, with Λ and G constant.

For the other values of n , we obtain

$$a = (n\beta t)^{1/n}, \quad (33)$$

$$Ht = 1/n, \quad (34)$$

$$\rho_m/\rho = n/3. \quad (35)$$

For $n = 2, 3$, we have decelerating solutions, leading to a too young universe and a too high matter density. For $n = 1$, it follows that

$$a = \beta t, \quad (36)$$

$$Ht = 1, \quad (37)$$

$$\rho_m/\rho = 1/3. \quad (38)$$

Therefore, we re-obtain a coasting expansion, with an acceptable age for the universe. Moreover, this time the corresponding relative matter density matches surprisingly well the observed value.

IV. CONCLUDING REMARKS

To conclude, let us obtain the variation rate of G , and the rate of matter production in this model.

For G we have used the evolution law

$$G = \frac{H}{8\pi\lambda}, \quad (39)$$

which leads to the relative variation rate

$$\dot{G}/G = -(1+q)H, \quad (40)$$

where $q = -a\ddot{a}/\dot{a}^2$ is the deceleration parameter.

For $n = 1$, we have $q = 0$, and so

$$\dot{G}/G = -H. \quad (41)$$

For the rate of matter production (coming from the decaying vacuum energy), we have (for $n = 1$)

$$\frac{1}{a^3} \frac{d}{dt}(\rho_m a^3) = 2\rho_m H. \quad (42)$$

Finally, a note on the variation of the vacuum energy in the present context. It is possible to show that the vacuum energy density and the corresponding cosmological “constant” are given by

$$\rho_\Lambda = (3-n)\lambda H = \frac{(3-n)}{8\pi} m^3 H, \quad (43)$$

$$\Lambda = 8\pi G \rho_\Lambda = (3-n)H^2, \quad (44)$$

leading to present values in agreement with observation.

The first equation is in accordance with a recent derivation by Schützhold [11], based on quantum field calculations in an expanding background. The second equation agrees with an ansatz originally proposed by Chen and Wu [2], and generalized later by Carvalho, Lima and Waga [3], and by Alcaniz and Maia [4]. Note that, in the realm of a constant G , those two equations would be incompatible to each other.

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